2011-2012
IB MYP YEAR 4

SUMMATIVE ASSESSMENT

Year 9 Mathematics (Extended)

Name: KWOK Chun Hei [9 Hope]

Date of task: **8th June, 2012** Time allowed: **1.5 hours (11:40 -13:10)**

Teacher: $\boxed{\textbf{Ms Li}}$ / **Mr Millard** / **Mr So**

Student's **Performance** in Different **Criteria**			
A	7	**C**	6

Instructions

- Read the instructions for all questions carefully.
- All work must be hand written.
- All work, steps and proper units must be shown.
- A non-electronic dictionary is allowed.
- Use of calculator is allowed.

Advice:

- Read the criteria descriptors and task-specific rubrics carefully before you start your work. This will give you a clear understanding of what is required and what a high quality piece of work for this task must include. This way you give yourself the best chance of achieving the highest levels in this task.

- This assessment task will be assessed on Criterion **A** & **C**.
  - For Criteria **A**, the questions are all assigned with levels;
  - Criterion **C** will be assessed as an overall impression on the presentation of work in this assessment.

Assessment Criteria

Criterion A: KNOWLEDGE AND UNDERSTANDING

Achievement level	Task Specific Rubric	IBO Published Descriptor
0	The student does not reach a standard described by any of the descriptors given below.	The student does not reach a standard described by any of the descriptors given below.
1–2 **Simple**	The student can solve **some** **simple** problems.	The student **generally** makes appropriate deductions when solving **simple** problems in **familiar** contexts.
3–4 **Complex**	The student can solve **most** **simple** and **some** **more complex** problems.	The student **generally** makes appropriate deductions when solving **more complex** problems in **familiar** contexts.
5–6 **Challenging**	The student can solve **some** **challenging** problem along with **all** different types of problems.	The student **generally** makes **appropriate** deductions when solving **challenging** problems in a **variety** of **familiar** contexts.
7–8 **Unfamiliar**	The student can solve **most** challenging and **most** **unfamiliar** problems along with **all** different types of problems.	The student **consistently** makes **appropriate** deductions when solving **challenging** problems in a **variety** of contexts including **unfamiliar** situations.

Criterion C: COMMUNICATION IN MATHEMATICS

Achievement level	Task Specific Rubric	IBO Published Descriptor
0	The student does not reach a standard described by any of the descriptors given below.	The student does not reach a standard described by any of the descriptors given below.
1–2	The student should be able to explain **some problems** step by step. The lines of reasoning are **difficult to follow**.	The student shows **basic** use of mathematical language **and/or** forms of mathematical representation. The lines of reasoning are **difficult to follow**.
3–4	The student should be able to explain **most problems** step by step. The lines of reasoning are **clear** though **not always** **logical** or **complete**.	The student shows **sufficient** use of mathematical language **and** forms of mathematical representation. The lines of reasoning are **clear** though not always **logical** or **complete**. The student moves between different forms of representation **with some success**.
5–6	The student should be able to explain **most problems** step by step. The lines of reasoning are **concise**, **logical** and **complete**. The student use **correct unit** in the questions.	The student shows **good** use of mathematical language **and** forms of mathematical representation. The lines of reasoning are **concise**, **logical** and **complete**. The student moves **effectively** between different forms of representation.

A. SIMPLE PROBLEMS

Suggested time allocation for Question **1 to 5** is **15 minutes**.

1. Given the points A (–1 , 2) and B = (2 , **k**), find the value(s) of **k** such that the **length of line AB is 5 units**.

By distance formula,

AB: $\sqrt{(-1-2)^2+(2-k)^2} = 5$

$\sqrt{9+(4-4k+k^2)} = 5$

$\sqrt{k^2-4k+13} = 5$

$\left(\sqrt{k^2-4k+13}\right)^2 = 5^2$

$k^2-4k+13 = 25$

$k^2-4k-12 = 0$

$(k-6)(k-2) = 0$

$k = 6, 2$

$\begin{matrix} k & & -6 \\ & \times & \\ k & & 2 \end{matrix}$

∴ The values of k is 6 and 2.

2. In the figure on the right, find the value of **b without using calculator**.

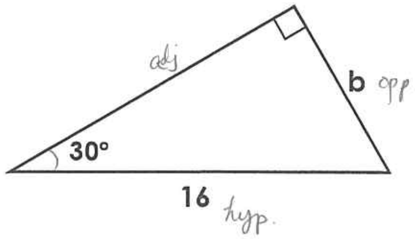
$\sin 30° = \frac{b}{16}$

$b = 16 \sin 30°$

$= 16 \times \frac{1}{2}$

$= 8$

∴ The value of b is 8.



3. Given that the equation of the line **L1** is $y - 2x = 4$, which of the following line(s) is/are **parallel to L1**? Which of the following line(s) has/have **negative y-intercepts**?

L2: $y = -2x + 4$ **L3:** $2y - 4x - 5 = 0$ **L4:** $-3y = 2x + 4$ **L5:** $6x - 9 = 3y$

Explain your answers by showing your calculations.

L_1: $y = 2x + 4$

L_2: $y = -2x + 4$

L_3: $y = 2x + \frac{5}{2}$

L_4: $y = -\frac{2}{3}x - \frac{4}{3}$

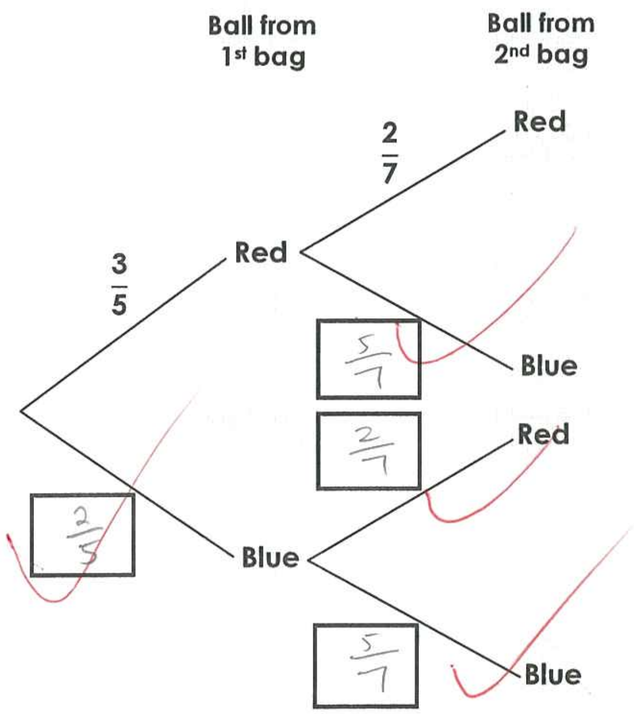
L_5: $y = 2x - 3$

∵ L_1, L_3 and L_5 have the same slope of +2,
∴ L_3 and L_5 are parallel to L_1.

∵ L_4 has a y-int of $-\frac{4}{3}$ and L_5 has a y-int of -3
∴ L_4 and L_5 have neg. y-int.

4. Loren has two bags. The **first** bag contains **3 red** balls and **2 blue** balls. The **second** bag contains **2 red** balls and **5 blue** balls. Loren takes **1 ball** at random from **each bag**.

(a) Complete the probability **tree diagram** by entering the **correct answers into the boxes.**



(b) Find the probability that Loren takes **two red balls.**

$P(2 \text{ red balls}) = \frac{3}{5} \times \frac{2}{7}$

$= \frac{6}{35}$

5. Evaluate the following **without using calculator.**

$$\sin^2 23° + \cos^2 23° - \frac{\sin 45°}{\cos 45°}$$

$\left(\sin^2 23° + \cos^2 23°\right) - \frac{\sin 45°}{\cos 45°}$

$= 1 - \left(\frac{\sqrt{2}}{2} \div \frac{\sqrt{2}}{2}\right)$

$= 1 - \left(\frac{\sqrt{2}}{2} \times \frac{2}{\sqrt{2}}\right)$

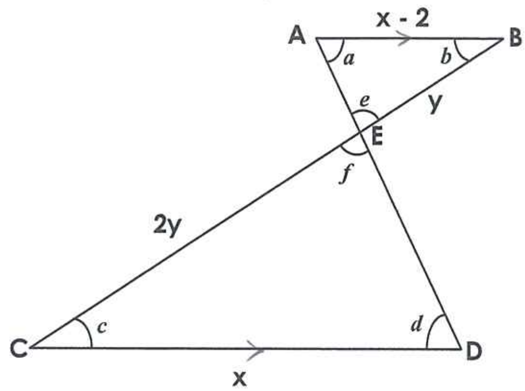
$= 1 - 1$

$= 0$

B. MORE COMPLEX PROBLEMS

Suggested time allocation for Question **6 to 9** is **25 minutes**.

6. In the figure below, the line AB is parallel to the line CD and some dimensions are shown in terms of x or y.



(a) Show that Δ **ABE** and Δ**DCE** are **similar**. State the reason(s) if necessary.

$e = f$ (vert. opp. ∠s)

$b = c$ (alt. ∠s, AB // CD)

$a = d$ (alt. ∠s, AB // CD)

∴ $\triangle ABE \sim \triangle DCE$ (equiangular)

(b) Find the value of **x**.

$\frac{AB}{CD} = \frac{BE}{CE}$

$\frac{x-2}{x} = \frac{y}{2y}$

$\frac{x-2}{x} = \frac{1}{2}$

$2(x-2) = x$

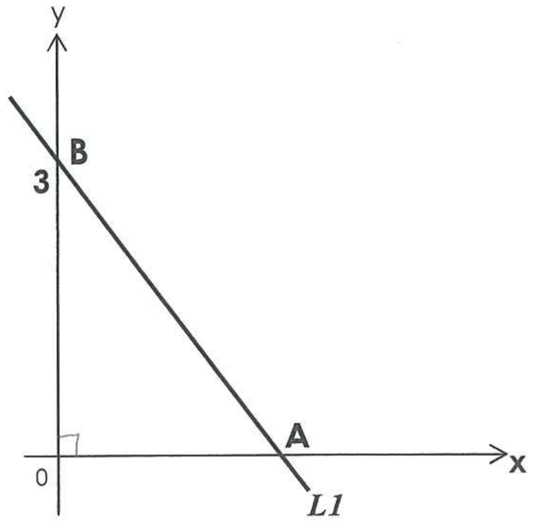
$2x - 4 = x$

$3x = 4$

$x = \frac{4}{3}$

∴ Value of x is $\frac{4}{3}$.

7. In the graph on the right, a line ***L1*** cuts the x-axis and y-axis at point **A** and **B** respectively. The y-intercept is 3.



(a) If the area of the triangle AOB is 3 square units, find the **equation** of ***L1***. Express your answer in **slope-intercept form**.

∵ B is 3 units from the origin,
and A is A units from the origin.

∵ $\frac{AB}{2} = 3$

$\frac{3A}{2} = 3$

$3A = 6$

$A = 2$

∵ Coordinates of B is (0,3)
Coordinates of A is (2,0)

∴ By two-point form

$\frac{y-3}{x-0} = \frac{3-0}{0-2}$ → $2(y-3) = -3x$

$\frac{y-3}{x} = -\frac{3}{2}$

$2y-6 = -3x$

$2y = -3x+6$

$y = -\frac{3}{2}x+3$

(b) If a line ***L2*** is **perpendicular** to ***L1*** and two lines intersect at point **D(4,-3)**, find the equation of ***L2***. Express your answer in **general form**.

$L_1: y = -\frac{3}{2}x+3$

∵ $L_1 \perp L_2$

∴ $m_{L_1} \times m_{L_2} = -1$

$-\frac{3}{2} \times m_{L_2} = -1$

$m_{L_2} = \frac{2}{3}$

By point-slope form,

$L_2: \frac{2}{3} = \frac{y-(-3)}{x-4}$

$\frac{2}{3} = \frac{y+3}{x-4}$

$3(y+3) = 2(x-4)$

$3y+9 = 2x-4$

$-2x+3y+\cancel{13} = 0$

8. In a certain dice game, the player throws **two** typical unbiased **six-faces dice** and receives **$5** if the sum is **7 or 11**, otherwise he or she **pays $2**.

(a) Calculate the probability of obtaining the **sum of 7 or 11** when you <u>**throw the two dice once**</u>.

Sum of throwing 2 dices

	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

$P(7 \text{ or } 11) = \frac{8}{36}$

$= \frac{2}{9}$

(b) If you play the game **18 times**, calculate the **amount of money** you <u>**expect to gain or lose**</u>.

$(1+2+3+4+5+6+7+8+9+10+11+12+13+14+15+16+17+18)\frac{2}{9}$

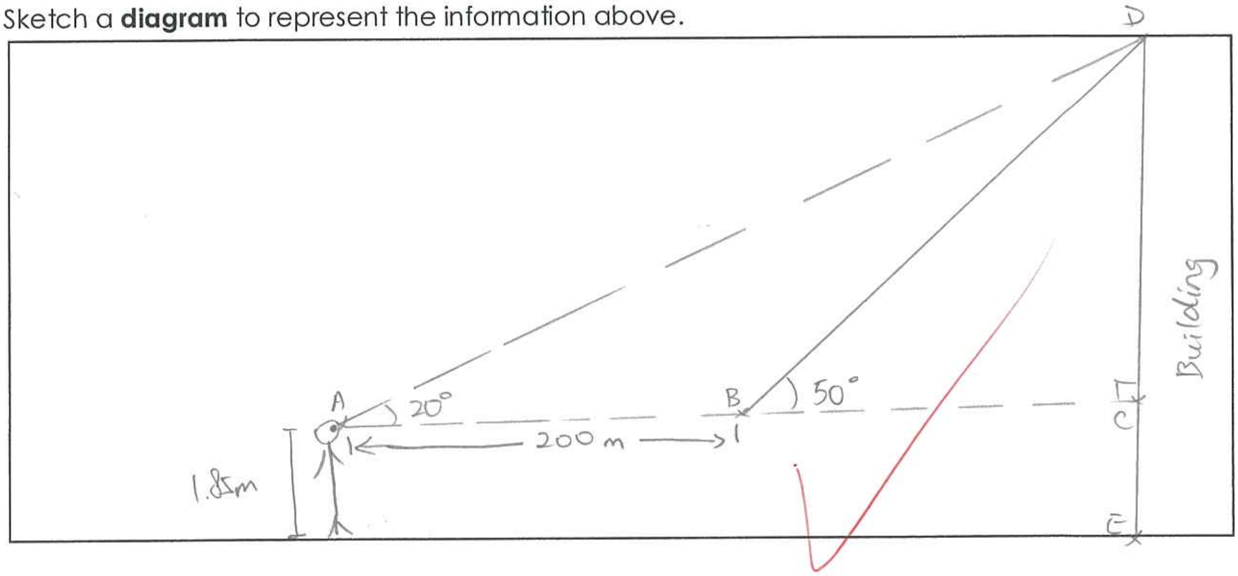
$= 171 \times \frac{2}{9}$

$= 38$

$\therefore$ It is expect to gain $38.

9. Mr Bolivar, a volunteer fireman who is 1.85 m tall, is running towards a burning building where there is a fire on the roof. Initially, his angle of elevation to the roof is 20°. He runs for 200 m and now his angle of elevation is 50°. Assume that the ground is horizontal and the building is vertical.

(a) Sketch a **diagram** to represent the information above.



(b) How tall is the building? Correct your answer to the **nearest meter**.

$\tan 50° = \frac{CD}{BC}$

$CD = BC \tan 50°$ — (1)

$\tan 20° = \frac{CD}{AC}$

$= \frac{CD}{AB + BC}$

$CD = (200 + BC)\tan 20°$ — (2)

$\therefore CD = BC \tan 50° = (200 + BC)\tan 20°$

$= 200 \tan 20° + BC \tan 20°$

$BC \tan 50° - BC \tan 20° = 200 \tan 20°$

$BC(\tan 50° - \tan 20°) = 200 \tan 20°$

$BC = \frac{200 \tan 20°}{\tan 50° - \tan 20°}$

$= 87.9$ (m) (corr. to 3 sig. fig.) — (3)

Sub. (3) into (1),

$CD = \frac{200 \tan 20°}{\tan 50° - \tan 20°} \times (\tan 50°)$

$= 105$ (m) (corr. to 3 sig. fig.)

$CE = CD + CE$

$= \frac{200 \tan 20° \tan 50°}{\tan 50° - \tan 20°} + 1.85m = 107$ (m) (corr. to 3 sig. fig.)

C. CHALLENGING PROBLEM

Suggested time allocation for Question **10 and 11** is **30 minutes**.

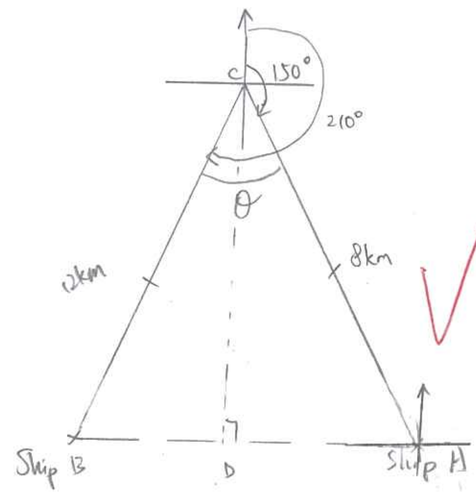
10. Ship **A** leaves the harbor H on a bearing **150°** with a speed of **40** km/hr. At the same time, Ship **B** leaves harbor H on a bearing **210°** with a speed of **40** km/hr.

(a) **After 12 minutes,** how far did ship A and ship B travel?

Ship A : distance = 40km/hr × 12 mins
= 40km/hr × 0.2 hr
= 8km

Ship B : distance = 40km/hr × 12 mins
= 40km/hr × 0.2 hr
= 8km

(b) Sketch a **diagram** to represent the information above **12 minutes after the two ships left the harbor.**



(c) Find the **true bearing from Ship A to Ship B** 12 minutes after they left the harbor.

270°

(d) Find the **distance between the two ships** 12 minutes after they left the harbor. Give your answer to the **nearest meter.**

$\theta = 210° - 150°$
$= 60°$

BC = AC = 8km (proven)
∠CDA = ∠CDB (by construction)
∠CBD = ∠CAD (base ∠s, isos. △)

∴ △CBD ≅ △CAD (A.S.A.)

∵ △CBD ≅ △CAD
∴ ∠ACD = ∠BCD = $\frac{\theta}{2}$ = 30°

$\sin 30° = \frac{AD}{AC}$
$= \frac{AD}{8km}$
AD = 8 sin 30°
= $8 \times \frac{1}{2}$
= 4

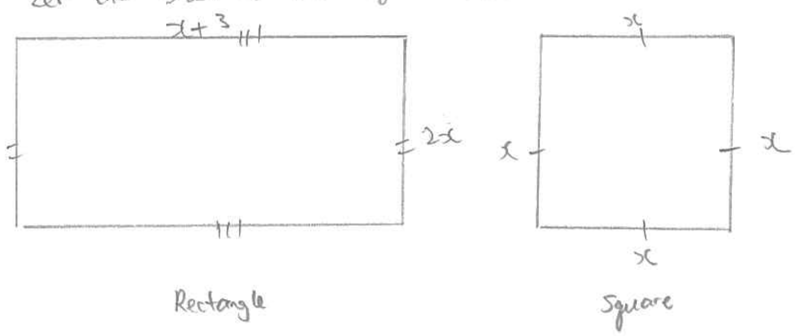
∵ △CBD ≅ CAD
∴ AD = BD (corr. sides, ≅ △s)
AD = BD = 4km
∴ AB = 8km

11. The properties of a rectangle and a square are given below:

- ◆ The length of the rectangle is 3 cm longer than the side of the square.
- ◆ The width of the rectangle is double the length of the side of the square.

If the **sum of their areas** is **24 cm²**, find the **dimensions** (that is, its length and width) of the rectangle.

Let the sides of the square be x.



Area of rectangle $= 2x(x+3)$

Area of square $= x^2$

$\because$ The sum of their areas $= 24\text{cm}^2$

$\therefore \quad 2x(x+3) + x^2 = 24$

$2x^2 + 6x + x^2 = 24$

$3x^2 + 6x - 24 = 0$

$3(x^2 + 2x - 8) = 0$

$x^2 + 2x - 8 = 0$

$(x+4)(x-2) = 0$

$x = -4, 2$

x	4
x	-2

$\because$ The rectangle and square is positive

$\therefore \quad x = 2$ (cm)

Sub. $x = 2$ into the dimensions of the rectangle,

Length $= 2 + 3 = 5$ (cm)

Width $= 2 \times 2 = 4$ (cm)

D. Unfamiliar problems (Suggested time allocation for Question **12 and 13** is **30 minutes**.)

12. At noon, Tom and Pete both park at the same starting point. Tom starts to ride his bike at 8 miles/hr. Two hours later, Pete starts after Tom on a bicycle at 12 miles/hr.

(a) How far will Tom have ridden before he is **overtaken by Pete**?

LCM: $4 \underline{| \; 12 \quad 8}$
$\quad\quad 3 \quad 2$

$4 \times 3 \times 2$

$= 24$ (miles)

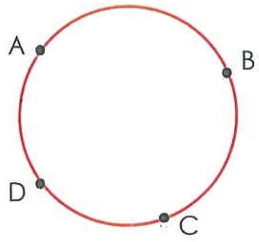
$\therefore$ Tom have ridden 24 miles before he is overtaken by Pete.

(b) At what time will Tom and Pete be **8 miles** apart?

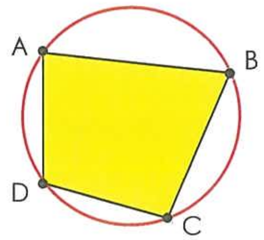
Time $= \frac{\text{Distance}}{\text{Speed}}$

13. Please read the following information and then do the proof on next page.

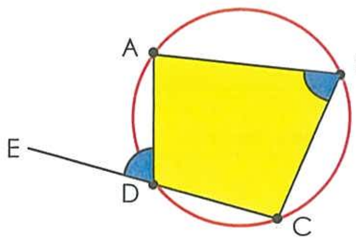
Points lie on the **same circle**, as the diagram below, are said to be **concyclic**. For example, A, B, C and D are **concyclic points.**



If the vertices of a **quadrilateral** lie on a **circle**, as the diagram below, then the quadrilateral is said to be **cyclic.** For example, ABCD is a **cyclic quadrilateral** since the vertices A, B, C and D lie on the circle.

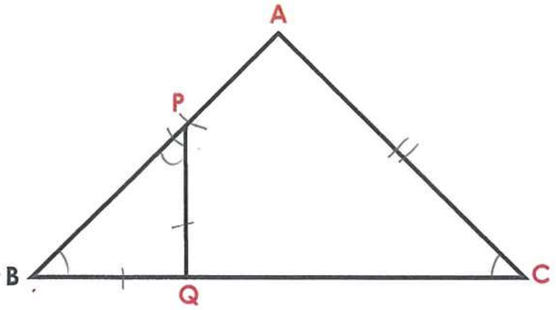


If the side CD is produced (i.e. extended) to E, as the diagram below, then $\angle$**ADE** is called the **exterior angle of the cyclic quadrilateral** ABCD, and $\angle$**ABC** is said to be the **interior opposite angle.**



Theorem: **If $\angle ADE = \angle ABC$, then A, B, C and D are concyclic. (ext.$\angle$, int. opp.$\angle$)**

In the figure below, **ΔABC** and **ΔBPQ** are **isosceles** triangles such that **AB = AC** and **BQ = PQ**.
Using the provided information about the concyclic points and cyclic quadrilateral, **prove** that **A, P, Q and C are concyclic.**



$\because$ $AB = AC$ (given)

$\therefore$ $\angle ABC = \angle ACB$ (base $\angle$s, isos. Δs)

$\because$ $PQ = BQ$ (given)

$\therefore$ $\angle BPQ = \angle QBP$ (base $\angle$s, isos. Δ)

$\because$ $\angle ABC = \angle QBP$ (common)

$\therefore$ $\angle ABC = \angle ACB = \angle BPQ = \angle QBP$

$\because$ $\angle ABC = \angle QBP$ (common)

$\angle BPQ = \angle ACB$ (proven)

$\angle BQP = \angle BAC$ ($\angle$ sum of Δ)

$\therefore$ $\Delta BPQ \sim \Delta BAC$ (equiangular)

$\because$ $\angle BQP = \angle BAC$ (proven)

$\therefore$ P, A, Q and C are concyclic.

End of Assessment